C4 VECTORS — Answers - Worksheet F

1 **a** $\overrightarrow{AB} = \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ -5 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$

$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$

b $2 - 2\lambda = 6 + a\mu$ (1)

$-1 + 4\lambda = -5 - 3\mu$ (2)

$-5 + \lambda = 1 + \mu$ (3)

(2) + 3×(3) $\Rightarrow$ $-16 + 7\lambda = -2$

$\lambda = 2,\ \mu = -4$

sub. (1) $2 - 2(2) = 6 + a(-4)$

$-2 = 6 - 4a$

$a = 2$

point of intersection: (−2, 7, −3)

2 **a** $\overrightarrow{BA} = (-4\mathbf{i} + 2\mathbf{j} - \mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k})$

$= -6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$

$\overrightarrow{BC} = (6\mathbf{i} + 4\mathbf{j} + \mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k})$

$= 4\mathbf{i} - \mathbf{j} + 8\mathbf{k}$

$|\overrightarrow{BA}| = \sqrt{36+9+36} = 9$

$|\overrightarrow{BC}| = \sqrt{16+1+64} = 9$

$\overrightarrow{BA}.\overrightarrow{BC} = -24 + 3 + 48 = 27$

$\cos(\angle ABC) = \frac{27}{9\times 9} = \frac{1}{3}$

b $\overrightarrow{AC} = (6\mathbf{i} + 4\mathbf{j} + \mathbf{k}) - (-4\mathbf{i} + 2\mathbf{j} - \mathbf{k})$

$= 10\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$

$= (-4\mathbf{i} + 2\mathbf{j} - \mathbf{k}) + \frac{1}{2}(10\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$

$= \mathbf{i} + 3\mathbf{j}$

c $\overrightarrow{BM} = (\mathbf{i} + 3\mathbf{j}) - (2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k})$

$= -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$

$\overrightarrow{BM}.\overrightarrow{AC} = (-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}).(10\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$

$= -10 - 4 + 14 = 0$

$\therefore$ BM perpendicular to AC

d $\angle ABC = \cos^{-1}\frac{1}{3} = 70.529$

isosceles triangle

$\therefore\ \angle ACB = \frac{1}{2}(180 - 70.529) = 54.7°$ (1dp)

3 **a** $\overrightarrow{AB} = \begin{pmatrix} 11 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

$\therefore \mathbf{r} = \begin{pmatrix} 9 \\ 5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

b $\overrightarrow{OC} = \begin{pmatrix} 9+2\lambda \\ 5+2\lambda \\ -3 \end{pmatrix}$

$\overrightarrow{OC} \cdot \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = 0$

$2(9 + 2\lambda) + 2(5 + 2\lambda) + 0 = 0$

$8\lambda + 28 = 0$

$\lambda = -\frac{7}{2}$, $\overrightarrow{OC} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$

c $OC = \sqrt{4+4+9} = \sqrt{17}$

$AC = \frac{7}{2} \left| \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \right| = 7\sqrt{1+1} = 7\sqrt{2}$

area $= \frac{1}{2} \times \sqrt{17} \times 7\sqrt{2} = 20.4$

d $AC = \frac{7}{2} AB$

$\therefore$ area OAB : area OAC = 2 : 7

4 **a** $|(7\mathbf{i} - 5\mathbf{j} - \mathbf{k})| = \sqrt{49+25+1} = 5\sqrt{3}$

$|(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})| = \sqrt{16+25+9} = 5\sqrt{2}$

$(7\mathbf{i} - 5\mathbf{j} - \mathbf{k}).(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = 28 + 25 - 3 = 50$

$\cos(\angle AOB) = \frac{50}{5\sqrt{3} \times 5\sqrt{2}} = \frac{2}{\sqrt{6}} = \frac{1}{3}\sqrt{6}$

b $\overrightarrow{AB} = (4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) - (7\mathbf{i} - 5\mathbf{j} - \mathbf{k})$

$= -3\mathbf{i} + 4\mathbf{k}$

$\overrightarrow{AB} \cdot \overrightarrow{OB} = (-3\mathbf{i} + 4\mathbf{k}).(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$

$= -12 + 0 + 12 = 0$

$\therefore$ AB perpendicular to OB

c $\overrightarrow{OC} = \frac{3}{2}(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$

$= 6\mathbf{i} - \frac{15}{2}\mathbf{j} + \frac{9}{2}\mathbf{k}$

$\overrightarrow{AC} = (6\mathbf{i} - \frac{15}{2}\mathbf{j} + \frac{9}{2}\mathbf{k}) - (7\mathbf{i} - 5\mathbf{j} - \mathbf{k})$

$= -\mathbf{i} - \frac{5}{2}\mathbf{j} + \frac{11}{2}\mathbf{k}$

$\overrightarrow{AC} \cdot \overrightarrow{OA} = (-\mathbf{i} - \frac{5}{2}\mathbf{j} + \frac{11}{2}\mathbf{k}).(7\mathbf{i} - 5\mathbf{j} - \mathbf{k})$

$= -7 + \frac{25}{2} - \frac{11}{2} = 0$

$\therefore$ AC perpendicular to OA

d $\angle CAO = 90°$

$\therefore$ $\angle ACO = 90° - \angle AOC$

$= 90° - \angle AOB$

$= 90° - \cos^{-1}(\frac{1}{3}\sqrt{6})$

$= 54.7°$